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A confidence limit for the empirical mode decomposition and Hilbert spectral analysis

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The confidence limit is a standard measure of the accuracy of the result in any statistical analysis. Most of the confidence limits are derived as follows. The data are first divided into subsections and then, under the ergodic assumption, the temporal mean is substituted for the ensemble mean. Next, the confidence limit is defined as a range of standard deviations from this mean. However, such a confidence limit is valid only for linear and stationary processes. Furthermore, in order for the ergodic assumption to be valid, the subsections have to be statistically independent. For non-stationary and nonlinear processes, such an analysis is no longer valid. The confidence limit of the method here termed EMD/HSA (for empirical mode decomposition/Hilbert spectral analysis) is introduced by using various adjustable stopping criteria in the sifting processes of the EMD step to generate a sample set of intrinsic mode functions (IMFs). The EMD technique acts as a pre-processor for HSA on the original data, producing a set of components (IMFs) from the original data that equal the original data when added back together. Each IMF represents a scale in the data, from smallest to largest. The ensemble mean and standard deviation of the IMF sample sets obtained with different stopping criteria are calculated, and these form a simple random sample set. The confidence limit for EMD/HSA is then defined as a range of standard deviations from the ensemble mean. Without evoking the ergodic assumption, subdivision of the data stream into short sections is unnecessary; hence, the results and the confidence limit retain the full-frequency resolution of the full dataset. This new confidence limit can be applied to the analysis of nonlinear and non-stationary processes by these new techniques. Data from length-of-day measurements and a particularly violent recent earthquake are used to demonstrate how the confidence limit is obtained and applied. By providing a confidence limit for this new approach, a stable range of stopping criteria for the decomposition or sifting phase

(EMD) has been established, making the results of the final processing with HSA, and the entire EMD/HSA method, more definitive.

Keywords: empirical mode decomposition; EMD/HSA; Hilbert spectral analysis; Hilbert–Huang transform (HHT); nonlinear data analysis; non-steady data analysis

1. Introduction

The two-step method of empirical mode decomposition (EMD) and Hilbert spectral analysis (HSA) introduced by Huang *et al*. (1998*a*, 1999*a*, hereafter referred to as H98 and H99) has proved to be a powerful procedure for analysing non-stationary and nonlinear data. During the years since its introduction, many applications have been found (Huang 2001; Huang *et al*. 1998*b*, 1999*b*, 2000, 2001; Gloersen & Huang 1999; Wu *et al*. 1999; Loh *et al*. 2001; Hu *et al*. 2002) that include analysing acoustic, biological, ocean, earthquake, climate and mechanical vibration data. As versatile as it has proved to be, the method still needs further clarifications and improvements. An aspect in need of development and clarification is the definition and analysis of a confidence limit for the resulting intrinsic mode functions (IMFs) and the Hilbert spectrum.

The confidence limit is a standard measure for results from statistical analysis. Ideally, it should be derived from an ensemble of observations and computed using the ensemble mean and standard deviation from this mean. Assuming the error is normally distributed, the confidence limit is usually defined as a range of values near this mean: one standard deviation is equivalent to 68%, and two standard deviations are equivalent to a 95% confidence limit.

6. Conclusions

Sifting is a general method of decomposing a given dataset into underlying scales of various sizes. By varying the chosen parameters in the sifting process, infinitely many IMF sets can be generated, or at least as many as needed. In this paper, these features of the EMD/HSA method have been used in a constructive way to examine data by introducing a statistical measure of the confidence limit from a single set of non-stationary and nonlinear data without invoking the ergodic assumption. With the help of the newly introduced confidence limit, a stable range of stopping criteria for the first step of the EMD/HSA method (the EMD-sifting operation) has also been established. This statistical measure has helped to make the EMD/HSA method more definitive. The scale parameters for the intermittence test are phenomenon dependent. Typically, a decomposition of the data (the first step, EMD) should be first made without the intermittence test. If mode mixing is clearly seen to occur, the scales should be determined from that result, and the selection made based on the time-scale for the intermittence test, so that each IMF can contain results of one narrow time-scale range. In the case of LOD, the decision is an easy one, for there are definite cycles. For other phenomena, it might not be so apparent. An indication of the scales present can also be determined from the marginal spectrum obtained through the EMD/HSA method applied without intermittence by identifying the peaks in that spectrum as an indication of the existence of relatively narrow band periodic variations. The scale parameter can be determined accordingly.

With these additions and improvements, we have increased the rigour of the EMD/HSA method, and thus also made it more robust and useful.